

Question # 1

1. Find the average rate of change of $f(x) = \sqrt{10x + 3}$ on the interval $[2, 5]$. Round to 2 decimal places.

$$f(5) = \sqrt{10(5) + 3} = \sqrt{53}$$

$$f(2) = \sqrt{10(2) + 3} = \sqrt{23}$$

$$\frac{f(5) - f(2)}{5 - 2} = \frac{\sqrt{53} - \sqrt{23}}{3}$$

$$= \mathbf{0.83}$$

Question # 2

2. Find the difference quotient of $f(x) = 11x^2 + 3$.

$$f(x + h) = 11(x + h)^2 + 3$$

$$= 11(x^2 + 2hx + h^2) + 3$$

$$= 11x^2 + 22hx + 11h^2 + 3$$

$$\frac{f(x + h) - f(x)}{h} = \frac{(\cancel{11x^2} + 22hx + \cancel{11h^2} + \cancel{3}) - (\cancel{11x^2} + \cancel{3})}{h}$$

$$= \frac{22hx + 11h^2}{h}$$

$$= \mathbf{22x + 11h}$$

Question # 3

3. Complete the following for $g(x) = \begin{cases} x + 1, & \text{if } -1 \leq x < 2 \\ 4 - x, & \text{if } 2 \leq x \leq 5. \end{cases}$

(a) Determine the domain of g .

Part-a

x goes from -1 until 5 so the domain is $[-1, 5]$

Part-b

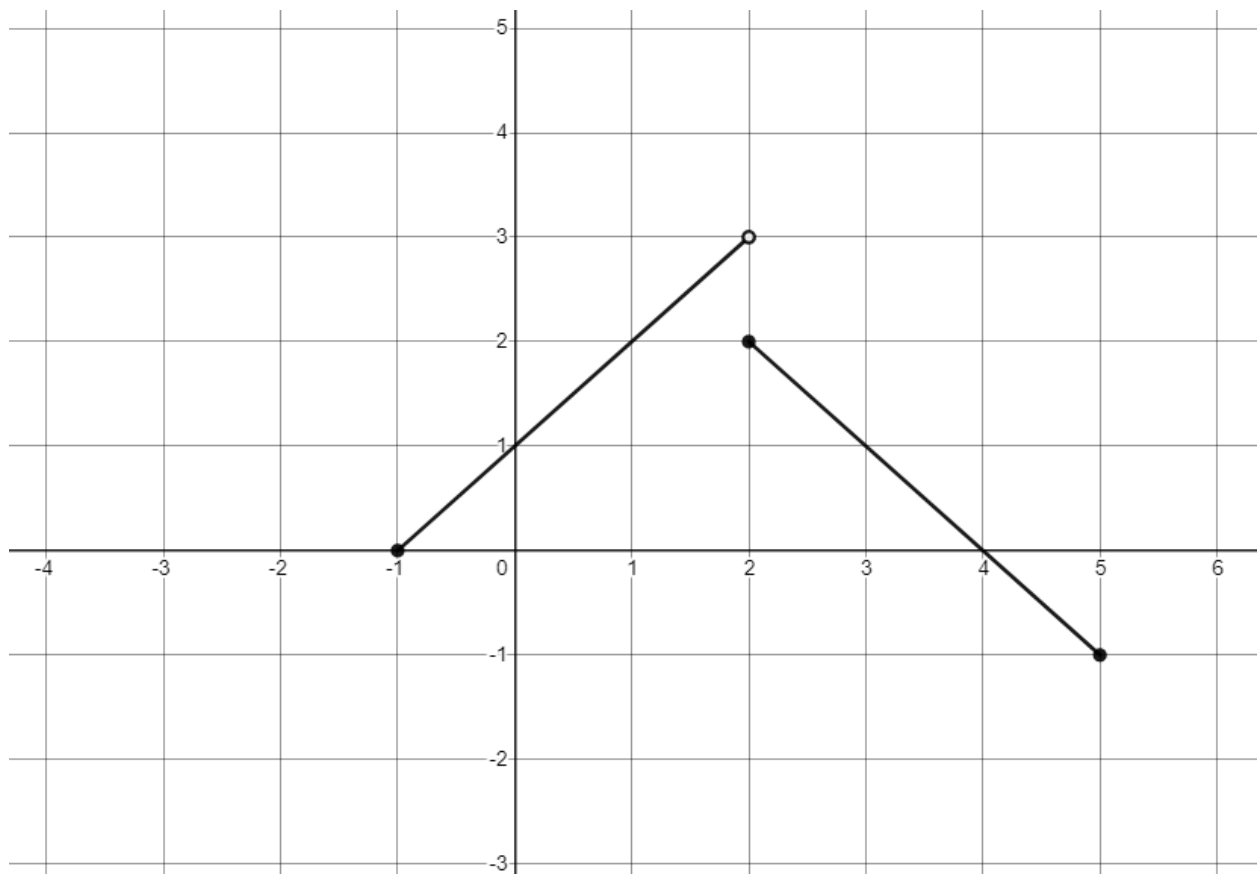
(b) Evaluate $g(2)$.

Use the second row because 2 is inside $2 \leq x \leq 5$

$$g(2) = 4 - 2 = 2$$

Part-c

(c) Graph g .



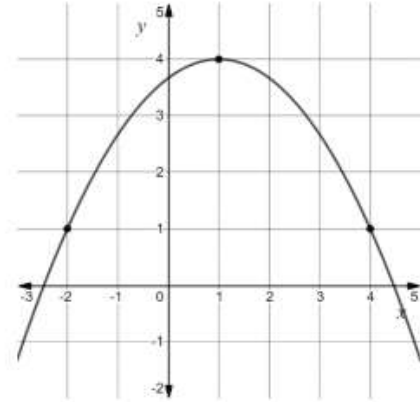
Part-d

(d) Is g continuous on its domain?

The function is NOT continuous because it is composed of two separate graphs. It has a jump discontinuity.

Question # 4

4. Use the graph (on the right) to model: $f(x) = a(x - h)^2 + k$.



The highest point or the vertex is at (1, 4) so $h = 1$ and $k = 4$

$$f(x) = a(x - 1)^2 + 4$$

(4, 1) is on the parabola so $f(4) = 1$ and we use it to solve for "a"

$$f(4) = a(4 - 1)^2 + 4$$

$$1 = 9a + 4$$

Subtract 4 from each side

$$-3 = 9a$$

Divide both sides by 9

$$a = \frac{-1}{3}$$

Therefore,

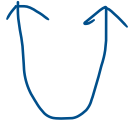
$$f(x) = \frac{-1}{3}(x - 1)^2 + 4$$

Question # 5

5. For $f(x) = 2x^2 - 12x + 23$, (a) sketch the concavity and identify the (b) vertex, (c) axis of symmetry, (d) range, (e) maximum or minimum value, (f) intervals where the function is increasing and where it is decreasing and (g) y-intercept. Then, (h) explain why the graph has no x-intercepts.

Part-a

The leading coefficient is $a = 2$ which is positive so the graph opens upward



Part-b

The coefficients are $a = 2$, $b = -12$, and $c = 23$

$$\frac{-b}{2a} = \frac{12}{4} = 3$$

$$f(3) = 2(3)^2 - 12(3) + 23 = 5$$

The vertex is at **(3, 5)**

Part-c

Since the vertex is at (3, 5), then $h = 3$ and the axis of symmetry has equation **$x = 3$**

Part-d

We know that the graph is a parabola that opens upward and the vertex or the lowest point is at (3, 5). Therefore, the range is **$[5, \infty)$**

Part-e

We know that the graph is a parabola that opens upward and the vertex or the lowest point is at (3, 5). Therefore, the minimum value is **5**

Part-f

Increasing on the interval $(3, \infty)$

Decreasing on the interval $(-\infty, 3)$

Part-g

$$f(0) = 2(0)^2 - 12(0) + 23 = 23$$

The y-intercept is 23 at coordinates (0, 23)

Part-h

Based on our calculated range from part-d which is $[5, \infty)$, the function cannot be equal to zero. Therefore, the graph has NO x-intercepts.

Question # 6

6. Simplify and write in standard form $\frac{12 \pm \sqrt{-60}}{10}$.

$$\sqrt{-60} = \sqrt{4}\sqrt{15}\sqrt{-1} = 2\sqrt{15}i$$

$$\frac{12 \pm 2\sqrt{15}i}{10}$$

Divide the numerator and denominator by 2

$$\frac{6 \pm \sqrt{15}i}{5}$$

We can express it in the form $a + bi$

$$\frac{6}{5} \pm \frac{\sqrt{15}}{5}i$$

Question # 7

7. Use your calculator to find a linear or quadratic function that best models the data below.

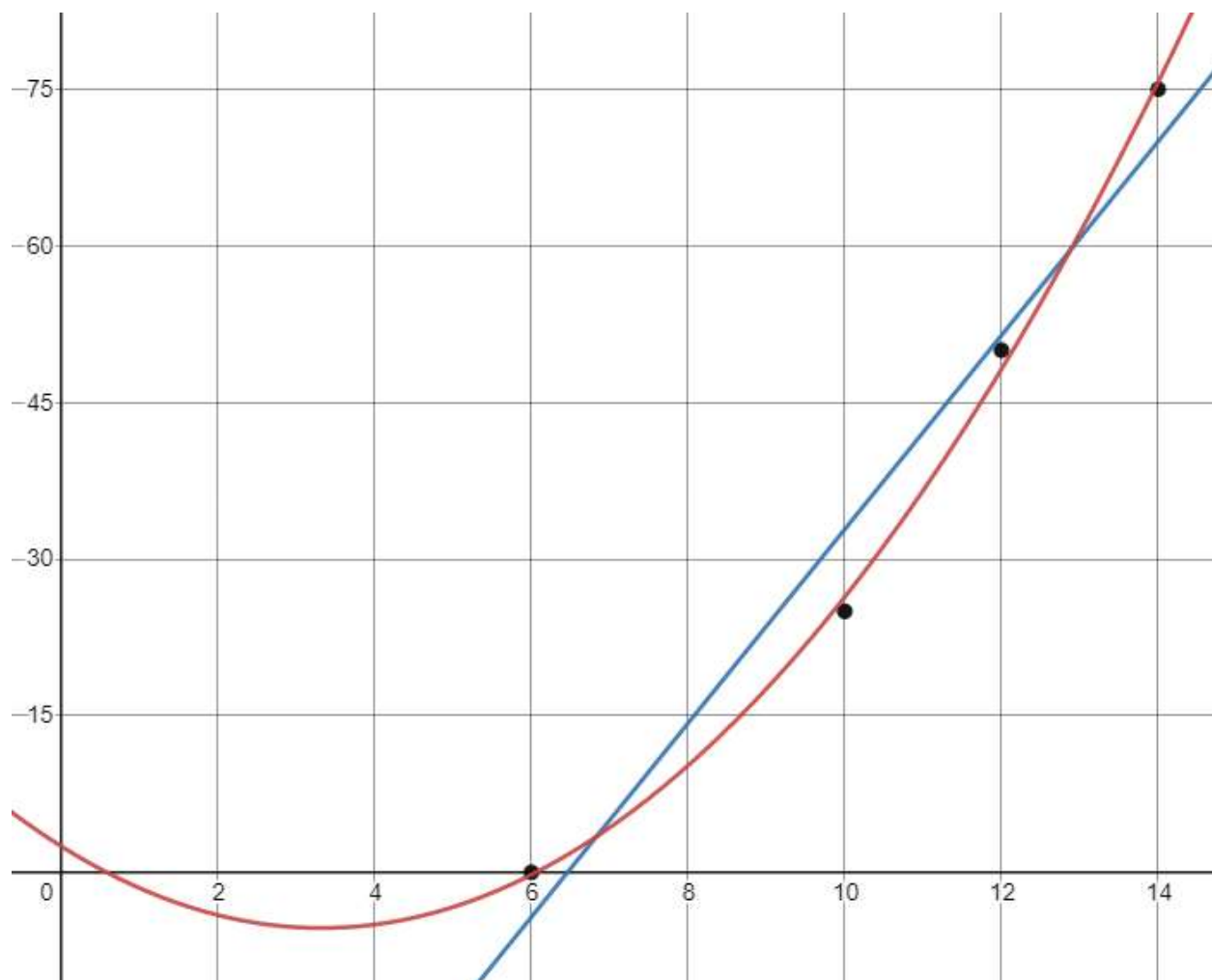
x	6	10	12	14
$f(x)$	0	25	50	75

(a) Circle the appropriate type of regression for the data set. Be sure to justify your answer.

Linear

Quadratic

Justification: Quadratic because the points are closer to the fitted quadratic function than the linear function.



Part-b

(b) Write down your regression equation. Round to 1 decimal place.

$$f(x) = 0.7x^2 - 4.7x + 2.5$$

(according to graphing calculator)

Part-c

(c) Use your model to predict $f(x)$ when $x = 5$.

$$f(5) = 0.7(5)^2 - 4.7(5) + 2.5 = -3.5$$

(there is a rounding error, if we use as many decimal places as possible, we get -3.32)

Part-d

(d) Did your answer involve interpolation or extrapolation?

Extrapolation because 5 is outside the range of values of x in the sample data.

Question # 8

8. Use the quadratic formula to find the x -intercepts of $f(x) = 3x^2 - 9x + 4$. Round to 1 decimal place.

The coefficients are $a = 3$, $b = -9$, and $c = 4$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{9 \pm \sqrt{81 - 4(3)(4)}}{6} \\&= \frac{9 \pm \sqrt{33}}{6}\end{aligned}$$

The x -intercepts to one decimal place are **(2.5, 0) and (0.5, 0)**

Question # 9

9. Find the domains of (a) $f(x) = \frac{x+1}{x-4}$ and (b) $g(x) = \frac{x+1}{x^2-8x+15}$. Write your solutions in interval notation.

Part-a

Equate the denominator to zero and exclude it from the domain

$$x - 4 = 0$$

$$x = 4$$

Therefore, the domain is all real numbers except 4

In interval notation, the domain is $(-\infty, 4) \cup (4, \infty)$

Part-b

Solve denominator = 0

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x - 3 = 0 \text{ or } x - 5 = 0$$

$$x = 3 \text{ or } 5$$

Therefore, the domain is $(-\infty, 3) \cup (3, 5) \cup (5, \infty)$

Question # 10

10. Write an equation for a function that has a graph with the shape of $y = |x|$, but is shifted right by 10 units, reflected about the x-axis, and shrunk vertically by a factor of $\frac{1}{4}$.

$$y = |x| \Rightarrow y = |x - 10| \quad (\text{shifted to right by 10 units})$$

$$\Rightarrow y = -|x - 10| \quad (\text{reflection in the x-axis})$$

$$\Rightarrow y = -\frac{1}{4}|x - 10| \quad (\text{vertically shrunk by a factor of } 1/4)$$

$$\text{Answer: } y = -\frac{1}{4}|x - 10|$$

Question # 11

11. Use the graph of $f(x)$ (on the left) to graph the transformation given (on the right).

$f(x)$

Graph $-2f(x+3)$

Use the rule $(x, y) \rightarrow (x-3, -2y)$

x	y	x-3	-2y
-4	-2	-7	4
-2	4	-5	-8
2	-4	-1	8
4	-2	1	4

Plot the points $(-7, 4)$, $(-5, -8)$, $(-1, 8)$, and $(1, 4)$ and connect them by line segments

